Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

1. Explain briefly why velocity fields must be conservative.

2. Evaluate $\int_C x^2\,\mathrm{d}x + yz\,\mathrm{d}y + (y^2/2)\,\mathrm{d}z$ along the line segment C joining $(0,0,0)$ and $(0,3,4)$.

3. For what values of b and c will $\mathbf{F} = (y^2 + 2czx)\mathbf{i} + y(bx + cz)\mathbf{j} + (y^2 + cx^2)\mathbf{k}$ be a gradient field?

4. (a) For constants G, m and M, verify that the **gravitational field** below is a gradient field:

$$\mathbf{F} = -GmM\frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{3/2}}$$

   (b) Find the potential function f.

   (c) Let P_1 and P_2 be points at distance s_1 and s_2 from the origin. Show that the work done by the gravitational field in moving a particle from P_1 to P_2 is

$$GmM\left(\frac{1}{s_2} - \frac{1}{s_1}\right).$$

5. Apply both forms of Green's Theorem to evaluate the integral

$$\oint_C (y^2\,\mathrm{d}x + x^2\,\mathrm{d}y)$$

   where C is the triangle bounded by $x = 0, x + y = 1, y = 0$.

6. **Area with Green's Theorem** The area of a region R bounded by C can be written as

$$\text{Area of } R = \iint_R \mathrm{d}y\,\mathrm{d}x = \iint_R \left(\frac{1}{2} + \frac{1}{2}\right)\mathrm{d}y\,\mathrm{d}x = \oint_C \frac{1}{2}x\,\mathrm{d}y - \frac{1}{2}y\,\mathrm{d}x.$$

   (a) Use this formula to find the area of the region of

      i. The ellipse $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (b\sin t)\mathbf{j}, \quad 0 \le t \le 2\pi$

      ii. The astroid $\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j}, \quad 0 \le t \le 2\pi$

   (b) Show that the area of the region R can also be written as

$$\text{Area of } R = \oint_C x\,\mathrm{d}y = -\oint_C y\,\mathrm{d}x.$$

7. What can be said about the curl component of a conservative two-dimensional vector field? Give reasons for your answer.