Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

1. **The Fundamental Theorem of Calculus** Prove the FTC for vector functions by using the theorem for scalar functions and the fact that *integration of vector functions can be done component by component* to show that if a vector function $\mathbf{r}(t)$ is continuous for $a \le t \le b$, then
$$\frac{\mathrm{d}}{\mathrm{d}t}\int_a^t \mathbf{r}(\tau)\,\mathrm{d}\tau = \mathbf{r}(t)$$
at every point t of (a,b). It can be shown that if $\mathbf{R}(t)$ is any antiderivative of $\mathbf{r}(t)$ on $[a,b]$, then any other antiderivative of $\mathbf{r}$ on $[a,b]$ equals $\mathbf{R}(t)+\mathbf{C}$ for some constant vector $\mathbf{C}$. Use this fact to show that
$$\int_a^b \mathbf{r}(t)\,\mathrm{d}t = \mathbf{R}(b) - \mathbf{R}(a).$$

2. Recall that the arc length parameter is given by the integral $s = \displaystyle\int_0^t |\mathbf{v}(\tau)|\,\mathrm{d}\tau$.
   (a) Find the arc length parameter of the curve $\mathbf{r}(t) = (1+2t)\mathbf{i} + (1+3t)\mathbf{j} + (6-6t)\mathbf{k}$.
   (b) Then find the length of the portion $-1 \le t \le 0$.
   (c) Verify that if $\mathbf{u}$ is a unit vector, then the arc length parameter along the line $\mathbf{r}(t) = P_0 + t\mathbf{u}$ from the point $P_0(x_0, y_0, z_0)$ where $t = 0$, is t itself.

3. **Center of Mass** The center of mass $(\bar{x}, \bar{y}, \bar{z})$ of a thin wire with density $\delta(x,y,z)$ is given by
$$\bar{x} = M_{yz}/M, \quad \bar{y} = M_{xz}/M, \quad \bar{z} = M_{xy}/M,$$
where $M = \displaystyle\int_C \delta\,\mathrm{d}s$, $M_{yz} = \displaystyle\int_C x\,\delta\,\mathrm{d}s$, $M_{xz} = \displaystyle\int_C y\,\delta\,\mathrm{d}s$, and $M_{xy} = \displaystyle\int_C z\,\delta\,\mathrm{d}s$.
Find the center of mass of a wire with density $\delta(x,y,z) = 15\sqrt{y+2}$ that lies along the curve $\mathbf{r} = (t^2-1)\mathbf{j} + 2t\mathbf{k}$, $-1 \le t \le 1$. Then sketch the curve and center of mass together.

4. Evaluate $\displaystyle\int_C (x-y)\,\mathrm{d}x + (x+y)\,\mathrm{d}y$ around the triangle with vertices $(0,0), (1,0)$, and $(0,1)$.

5. Give a formula $\mathbf{F} = M(x,y)\mathbf{i} + N(x,y)\mathbf{j}$ for the vector field in the plane that has the property below and find the *circulation* and *flux* done by $\mathbf{F}$ along the circle $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$, $0 \le t \le 2\pi$ and the ellipse $\mathbf{r}(t) = (\cos t)\mathbf{i} + (4\sin t)\mathbf{j}$, $0 \le t \le 2\pi$.
   (a) $\mathbf{F}$ points toward the origin with magnitude inversely proportional to the square of the distance from (x,y) to the origin. (The field is undefined at $(0,0)$.)
   (b) $\mathbf{F} = \mathbf{0}$ at $(0,0)$ and that at any other point (a,b), $\mathbf{F}$ is tangent to the circle $x^2+y^2 = a^2+b^2$ and points in the clockwise direction with magnitude $|\mathbf{F}| = \sqrt{a^2+b^2}$.

6. Suppose that $f(t)$ is differentiable and positive for $a \le t \le b$. Let C be the path $\mathbf{r}(t) = t\mathbf{i} + f(t)\mathbf{j}$, $a \le t \le b$, and $\mathbf{F} = y\mathbf{i}$. Is there any relation between the value of the work integral $\int_C \mathbf{F}\cdot\mathrm{d}\mathbf{r}$ and the area of the region bounded by the t-axis, the graph of f, and the lines $t = a$ and $t = b$? Give reasons for your answer.

7. let C be the ellipse in which the plane $2x + 3y - z = 0$ meets the cylinder $x^2 + y^2 = 12$. Show, using problem 1 and without evaluating either line integral directly, that the circulation of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ around C in either direction is zero.

8. Suppose that $\mathbf{F} = \nabla(xyz)$ is a gradient field. Find the line integral of $\mathbf{F}$ from $P(0,0,0)$ to $Q(1,1,1)$ along the following paths:
   (a) Straight line segment from P to Q
   (b) Two straight line segments first from P to $R(1,1,0)$ then from R to Q.
   (c) The path of intersection of the paraboloid $z = \frac{1}{2}(x^2+y^2)$ and the cylinder $y = x$. (*Hint*: Use $t = x$ as a parameter.)

   What have you noticed? We will learn more about this on Wednesday. =)