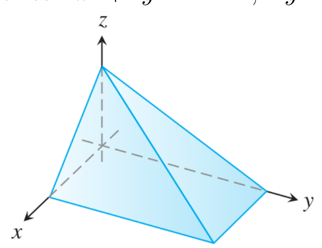
Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

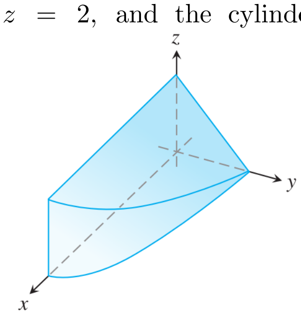
1. Let D be the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Write six different triple iterated integrals for the volume of D. Evaluate one of the integrals.

2. Sketch the region whose volume is represented by the integral $\displaystyle\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} \mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$. And rewrite five other integrals representing the same volume.

3. Use triple integration to find the volume of the following regions:
   (a) The region in the first octant bounded by the coordinate planes and the planes $x + y = 1$, $y + 2z = 2$.

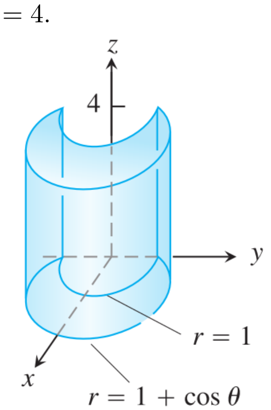
   (b) The region in the first octant bounded by the coordinate planes, the plane $y + z = 2$, and the cylinder $x = 4 - y^2$.

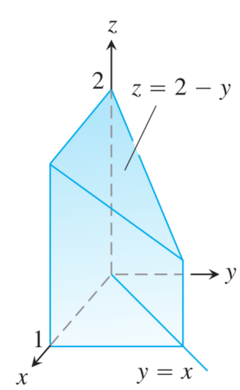
4. Evaluate $\displaystyle\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$ by changing the order of integration in an appropriate way.

5. Give the limits of integration for evaluating the integral $\displaystyle\iiint f(r,\theta,z)\,\mathrm{d}z\,r\,\mathrm{d}r\,\mathrm{d}\theta$ as an iterated integral over the region that is bounded below by the plane $z = 0$, on the side by the cylinder $r = \cos\theta$, and on top by the paraboloid $z = 3r^2$.

6. Set up the iterated integral for finding the volume of the solids below in cylindrical coordinates over region D.
   (a) D is the solid right cylinder whose base is the region in the xy-plane that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$ and whose top lies in the plane $z = 4$.

   (b) D is the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = 2 - y$.

7. (Bonus) The **centroid** $(\bar{x}, \bar{y})$ of a two dimensional region R is found by

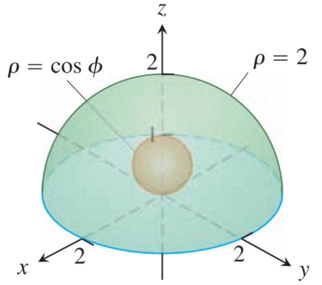
$$\bar{x} = \frac{M_y}{M} \quad \text{and} \quad \bar{y} = \frac{M_x}{M}$$

where $M = \iint\limits_R \mathrm{d}A$, $M_y = \iint\limits_R x\,\mathrm{d}A$, and $M_x = \iint\limits_R y\,\mathrm{d}A$.

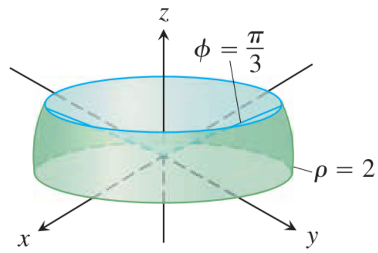
Use this to prove that the **centroid** of a triangle is the intersection of the three medians. (*Hint:* Without loss of generality, suppose the triangle's vertices are $(0,0), (a,b)$ and $(c,0)$ for all non-zero constants a, b and c. Can you extend this to find the **centroid** of a three-dimensional solid? For instance, the centroid of a tetrahedron?

8. Set up the iterated integral for finding the volume of the solids below in spherical coordinates, and evaluate the integral.

   (a) The solid between the sphere $\rho = \cos\phi$ and the hemisphere $\rho = 2, z \geq 0$.



   (b) The solid bounded below by the xy-plane, on the sides by the sphere $\rho = 2$, and above by the cone $\phi = \pi/3$.



9. **Substitution in double integrals** Suppose we want to evaluate the double integral

$$\iint\limits_R (2x^2 - xy - y^2)\,\mathrm{d}x\,\mathrm{d}y$$

where R is the region in the first quadrant bounded by the lines $y = -2x+4, y = -2x+7, y = x-2$ and $y = x+1$ by making a substitution

$$u = x - y, \quad v = 2x + y.$$

   (a) Sketch the region R in the xy plane.

   (b) Sketch the region G in the uv plane. (*Hint:* First solve for x and y in terms of u and v. Then rewrite the boundaries of R in terms of u and v.)

   (c) What is the ratio of the area of R over the area of G?

   (d) Find the value of the Jacobian $\partial(x,y)/\partial(u,v)$ and verify that it is the same as part (c).

10. **Substitution in triple integrals**

   (a) Let D be the region in xyz-space defined by the inequalities

$$1 \leq x \leq 2, \;\; 0 \leq xy \leq 2, \;\; 0 \leq z \leq 1.$$

   Evaluate $\iiint\limits_D (x^2y + 3xyz)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ by applying the transformation

$$u = x, \;\; v = xy, \;\; w = 3z$$

   and integrating over an appropriate region G in the uvw-space.

   (b) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

   by letting $x = au, y = bv$ and $z = cw$. Then find the volume of an appropriate region in uvw-space.