Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

1. For the following integrals, sketch the region of integration, reverse the order of integration, and evaluate the integral.

   (a) $\displaystyle\int_0^\infty\int_0^\infty x\mathrm{e}^{-(x+2y)}\,\mathrm{d}x\,\mathrm{d}y$

   (b) $\displaystyle\int_0^{\pi/4}\int_{\sin x}^{\cos x}\mathrm{d}y\,\mathrm{d}x$

   (c) $\displaystyle\int_{-1}^{2}\int_{y^2}^{y+2}\mathrm{d}x\,\mathrm{d}y$

   (d) $\displaystyle\int_0^2\int_{x^2-4}^{0}\mathrm{d}y\,\mathrm{d}x+\int_0^4\int_0^{\sqrt{x}}\mathrm{d}y\,\mathrm{d}x$

2. Sketch the solid whose volume is given by the integral

$$\int_0^4\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}}\sqrt{25-x^2-y^2}\,\mathrm{d}x\,\mathrm{d}y.$$

3. A solid right cylinder (z variable is free) has its base R in the xy-plane and is bounded above by the paraboloid $z=x^2+y^2$. The volume is given by

$$V=\int_0^1\int_0^y(x^2+y^2)\,\mathrm{d}x\,\mathrm{d}y+\int_1^2\int_0^{2-y}(x^2+y^2)\,\mathrm{d}x\,\mathrm{d}y.$$

   Sketch the base region R and express the cylinder's volume as a single iterated integral with the order of integration reversed. Then evaluate the integral to find the volume.

4. Evaluate the definite integral

$$\int_0^1\tan^{-1}x\,\mathrm{d}x$$

   using three methods: **(1)** by substitution **(2)** by parts, **(3)** by re-writing the integral as a double integral, reverse the order of integration, and integrate it.

5. Find the area of region common to the interiors of the cardioids $r=1+\cos\theta$ and $r=1-\cos\theta$.

6. (a) Change the Cartesian integral $\displaystyle\int_{-1}^{1}\int_0^{\sqrt{1-x^2}}\mathrm{d}y\,\mathrm{d}x$ into an equivalent polar integral.

   (b) Change the polar integral $\displaystyle\int_0^{\pi/2}\int_0^1 r^3\sin\theta\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$ into a Cartesian integral.

7. Integrate $f(x,y)=[\ln(x^2+y^2)]/\sqrt{x^2+y^2}$ over the region $1\le x^2+y^2\le \mathrm{e}$ by converting it to a polar integral.

8. Use polar integration to find

$$\int_0^\infty\int_0^\infty\frac{1}{(1+x^2+y^2)^2}\,\mathrm{d}x\,\mathrm{d}y.$$

9. Use the double integral in polar coordinates to derive the formula

$$A=\int_\alpha^\beta\frac{1}{2}r^2\,\mathrm{d}\theta$$

   for the area of the fan-shaped region between the origin and the polar curve $r=f(\theta),\alpha\le\theta\le\beta$.

10. Use the concept of the Jacobian determinant to explain the stretching and shrinking factor in evaluating a definite integral by u-substitution:

$$\int_a^b f(g(x))\,\mathrm{d}x=\int_{u=g(a)}^{u=g(b)}f(u)\,\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$