Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

1. Find the absolute maxima and minima of the function $f(x, y) = x^2 + y^2$ on the closed triangular plate bounded by the lines $x = 0, y = 0, y + 2x = 2$ in the first quadrant. (First consider the interior points by using the first derivative test. Then consider the boundary points.)

2. Find two numbers a and b with $a \leq b$ that maximizes the integral
$$\int_a^b (6 - x - x^2)\,\mathrm{d}x.$$

3. Find the critical point of
$$f(x, y) = xy + 2x - \ln x^2 y$$
in the open first quadrant ($x > 0, y > 0$) and show that f takes on a minimum there.

4. Find the maxima, minima and saddle points of $f(x, y)$, if any, given that $f_x = 2x - 4y$ and $f_y = 2y - 4x$.

5. Find the minimum distance from the point $(2, -1, 1)$ to the plane $x + y - z = 2$ by using three methods: (1) vector (2) substitution (3) Lagrange multiplier method.

6. Find the absolute maximum and minimum values of the function $f(x, y) = x + y$ on the semicircle $x^2 + y^2 = 4, y \geq 0$ using three methods: (1) substitution (2) Lagrange multiplier (3) use the parametric equations $x = 2\cos t, y = 2\sin t$ and treat f as a function of the single variable t and use the Chain Rule to find where $\mathrm{d}f/\mathrm{d}t$ is zero.

7. **Least squares and regression line** Given an array of data points $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$, we try to fit a line $y = mx + b$ through it by minimizing the sum of the squares of the vertical distances from the points to the line. This means finding the values of m and b that minimize the value of
$$w = (mx_1 + b - y_1)^2 + \cdots + (mx_n + b - y_n)^2.$$
Show that the values of m and b are
$$m = \frac{S_x S_y - n S_{xy}}{S_x^2 - n S_{x^2}}$$
$$b = \frac{1}{n}(S_y - m S_x)$$
where $S_x = \sum x_k, S_y = \sum y_k, S_{xy} = \sum x_k y_k$ and $S_{x^2} = \sum x_k^2$.

8. **Utility maximization** In economics, the *utility* of amounts x and y of two capital goods G_1 and G_2 is measured by a function $U(x, y)$. For example, G_1 and G_2 might be two chemicals a pharmaceutical company needs to produce a certain drug and $U(x, y)$ measure the gain from manufacturing this product. If G_1 costs a dollars per kilogram, and G_2 costs b dollar per kilogram, and the total amount allocated for the purchase of G_1 and G_2 together is c dollars. (The **budget constraint** is $ax + by = c$. Note that $\mathbf{p} = \langle a, b\rangle$ is called the **price vector**, and the budget constraint is usually denoted by $\mathbf{p} \cdot \mathbf{x} = c$.) Economists are interested in the amount x and y that maximizes the utility function. Suppose that the utility function is given by
$$U(x, y) = xy + 2x.$$
Find the maximum value of U subject to the constraint $2x + y = 30$.