Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

1. Use the limit definition of partial derivatives to compute the partial derivatives f_x and f_y of the function $f(x, y) = 1 - x + y - 3x^2y$, at $(1, 2)$.

2. For the following functions, find $\partial f/\partial x$ and $\partial f/\partial y$.

   (a) $f(x, y) = (xy - 1)^2$
   (b) $f(x, y) \sin^2(x - 3y)$
   (c) $f(x, y) = \tan^{-1}(y/x)$
   (d) $f(x, y) = \mathrm{e}^{xy} \ln y$
   (e) $f(x, y, z) = yz \ln(xy)$
   (f) $f(x, y, z) = \mathrm{e}^{-(x^2+y^2+z^2)}$

3. For the following functions, find f_{xx}, f_{yy} and f_{xy}.

   (a) $f(x, y) = x + y + xy$
   (b) $f(x, y) = \sin xy$
   (c) $f(x, y) = y\mathrm{e}^{x^2-y}$
   (d) $f(x, y) = \ln(x + y)$

4. The fifth-order partial derivative $\partial^5 f/\partial x^2 \partial y^3$ is zero for each of the following functions. Determine which variable would you differentiate first, x or y, without actually calculating it.

   (a) $f(x, y) = y^2x^4\mathrm{e}^x + 2$
   (b) $f(x, y) = y^2 + y(\sin x - x^4)$
   (c) $f(x, y) = x^2 + 5xy + \sin x + 7\mathrm{e}^x$
   (d) $f(x, y)x\mathrm{e}^{y^2/2}$.

5. find the value of $\partial x/\partial z$ at the point $(1, -1, -3)$ fi the equation

$$xz + y \ln x - x^2 + 4 = 0$$

   defines x as a function of the two independent variables y and z and the partial derivative exists.

6. Express v_x in terms of u and y if the equation $x = v \ln u$ and $y = u \ln v$ define u and v as functions of the independent variables x and y, and if v_x exists. (*Hint:* Differentiate both equations with respect to x and solve for v_x by eliminating u_x.)

7. Let $f(x, y) = \begin{cases} y^3, & y \geq 0 \\ -y^2, & x < 0. \end{cases}$

   Find f_x, f_y, f_{xy} and f_{yx}, and state the domain for each partial derivative.

8. Draw a branch diagram and write a Chain Rule formula for each derivative:

   (a) $\dfrac{\mathrm{d}z}{\mathrm{d}t}$ for $z = f(x, y)$, $x = g(t), y = h(t)$
   (b) $\dfrac{\partial y}{\partial r}$ for $y = f(u)$, $u = g(r, s)$
   (c) $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ for $w = f(x, y)$, $x = g(r)$, $y = h(s)$
   (d) $\dfrac{\partial w}{\partial u}$ and $\dfrac{\partial w}{\partial v}$ for $w = h(x, y, z)$, $z = f(u, v), y = g(u, v), z = k(u, v)$

9. Use the Chain Rule to find the following derivatives

   (a) $\dfrac{\mathrm{d}w}{\mathrm{d}t}$ for $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$
   (b) $\dfrac{\mathrm{d}w}{\mathrm{d}t}$ for $w = 2y\mathrm{e}^x - \ln z$, $x = \ln(t^2 + 1)$, $y = \tan^{-1} t$, $z = \mathrm{e}^t$
   (c) $\dfrac{\partial z}{\partial u}$ for $z = 4\mathrm{e}^x \ln y$, $x = \ln(u \cos v)$, $y = u \sin v$
   (d) $\dfrac{\partial w}{\partial v}$ for $u = \dfrac{p - q}{q - r}$, $p = x + y + z$, $q = x - y + z$, $r = x + y - z$