Chinese name: ______________ English name: ______________ Homeroom: _______ Math Class: _______

1. (a) Express the area A of the cross-section cut from the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

by the plane $z = c$ as a function of c. (The area of an ellipse with semiaxes a and b is πab.)

(b) Use slices perpendicular to the z-axis to find the volume of the ellipsoid in part (a).

(c) Now find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Does your formula give the volume of a sphere of radius a if $a = b = c$?

2. Show that the volume of the segment cut from the paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

by the plane $z = h$ equals half the segment's base times its altitude.

3. For the following functions, **(1)** find the function's domain, **(2)** find the function's range, **(3)** describe the function's level curves, **(4)** find the boundary of the function's domain, **(5)** determine if the domain is an open region, a closed region, or neither, and **(6)** decide if the domain is bounded or unbounded.

(a) $f(x, y) = y - x$

(b) $f(x, y) = \sqrt{y - x}$

(c) $f(x, y) = 4x^2 + 9y^2$

(d) $f(x, y) = xy$

(e) $f(x, y) = \ln(x^2 + y^2)$

(f) $f(x, y) = \frac{y}{x^2}$

(g) $f(x, y) = \mathrm{e}^{-x^2-y^2}$

(h) $f(x, y) = \sqrt{9 - x^2 - y^2}$

4. Find an equation for the level surface of the function

$$f(x, y, z) = \sqrt{x - y} - \ln z$$

through the point $(3, -1, 1)$.

5. For the following problems, find and sketch the domain of f. Then find an equation for the level curve or surface of the function passing through the given point

(a) $f(x, y) = \sum_{n=0}^{\infty} \left(\frac{x}{y}\right)^n, \quad (1, 2)$

(b) $g(x, y, z) = \sum_{n=0}^{\infty} \frac{(x + y)^n}{n! z^n}, \quad (\ln 4, \ln 9, 2)$

(c) $f(x, y) = \int_x^y \frac{\mathrm{d}\theta}{\sqrt{1 - \theta^2}}, \quad (0, 1)$

(d) $g(x, y, z) = \int_x^y \frac{\mathrm{d}t}{1 + t^2} + \int_0^z \frac{\mathrm{d}\theta}{\sqrt{4 - \theta^2}}, \quad (0, 1, \sqrt{3})$

6. By considering different paths of approach, show that following functions have no limit as $(x, y) \to (0, 0)$.

(a) $f(x, y) = -\frac{x}{\sqrt{x^2 + y^2}}$

(b) $f(x, y) = \frac{x^4}{x^4 + y^2}$

(c) $f(x, y) = \frac{xy}{|xy|}$

(d) $f(x, y) = \frac{x^2 + y}{y}$