Chinese name: _____ English name: _____ Homeroom: _____ Date: _____

1. Show that the function $f(x,y) = -\dfrac{x}{\sqrt{x^2+y^2}}$ has no limit as $(x,y) \to (0,0)$. [5 *marks*]

2. Let $f(x,y) = \begin{cases} \sqrt{x} & x \geq 0 \\ x^2 & x < 0. \end{cases}$ Find f_x with its domain specified. [5 *marks*]

3. Use the Chain Rule and the fact that $\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_a^b f(t,x)\,\mathrm{d}t = \int_a^b \frac{\partial}{\partial x} f(t,x)\,\mathrm{d}t$, find the derivative of the function $F(x) = \displaystyle\int_{x^2}^1 \sqrt{t^3+x^2}\,\mathrm{d}t$. [8 *marks*]

4. Estimate how much will $f(x, y, z) = \mathrm{e}^x \cos yz$ change as the point $P(x, y, z)$ moves from the origin a distance of $\mathrm{d}s = 0.1$ unit in the direction of $2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. [*5 marks*]

5. Find the absolute maxima and minima of the function $f(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by $x = 0, y = 4, y = x$. [*9 marks*]

6. Find the point on $x^2 + y^2 + z^2 = 25$ where $f(x, y, z) = x + 2y + 3z$ is maximized. [*9 marks*]

7. Let D be the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$. Write triple iterated integrals in the order $\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$ and $\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$. (No need to evaluate.) [*6 marks*]

8. Find the volume of the region whose base is the region between the circles $r = \cos\theta$ and $r = 2\cos\theta$ and whose top lies in the plane $z = 3 - y$. [*8 marks*]

9. (a) Find the area of the cardiod $r = 1 + \cos\theta$ using a double integral in polar coordinates. [*4 marks*]

(b) Now imagine the cardiod $r = 1 + \cos\theta$ is in the zx-plane (θ is measured with respect to the z-axis) and is revolved 360° around the z axis. Find the resulting surface in terms of the spherical coordinates ρ, ϕ, θ. [*4 marks*]

(c) Find the volume of the cardiod of revolution from above using spherical integration. [*7 marks*]

10. Evaluate the integral $\iint_R 2(x-y)\,\mathrm{d}x\,\mathrm{d}y$ where R is the parallelogram in the xy-plane with boundaries $x = -3, x = 0, y = x$ and $y = x + 1$. by making the substitution $u = 2x - 3y, v = -x + y$. Sketch both regions R and the transformed region G. [*10 marks*]